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Controller Reduction for Effective Interdisciplinary Design of Active Structures

Mark J. Balas
Ralph Quan

Center for Space Structures and Controls and
Department of Aerospace Engineering Sciences
Campus Box 429
University of Colorado
Boulder, Colorado

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Introduction:

Control problems of large aerospace structures are intrinsically interdisciplinary and require strategies which address the complete interaction between flexible structures, electromechanical actuators and sensors, and feedback control algorithms. In this paper we would like to survey our current research and future directions which will require an interdisciplinary team effort in dynamics, control and optimization of such structures.

It is generally agreed that the dynamics of space structures require large-scale discrete modeling, resulting in thousands of discrete unknowns. Proven control strategies, on the other hand, employ a low-order controller that is based on a reduced-order model of structures. Integration of such low-order controllers and large-scale dynamics models often leads to serious deterioration of the closed-loop stability margin and even instability. To alleviate this stability deterioration while low-order controllers remain effective, we have investigated the following approach:

(a) Retain low-order controllers based on reduced-order models of structures as the basic control strategy;

(b) Introduce a compensator that will directly account for the deterioration of stability margin due to controller-structure integration;

(c) Assess overall performance of the integrated control-structure system by developing measures of suboptimality in the the combination of (a) and (b).

The benefits of this approach include:

(1) Simplicity in the design of basic controllers, thus facilitating the optimization of structure-control interactions;

(2) Increased understanding of the roles of the compensator so as to modify the structure as well as the basic controller, if necessary, for improved performance;

(3) Adaptability to localize controllers by viewing the compensator as a systems integration filter.

We have demonstrated the above approach in the active control design of a simulation of a three-dimensional truss beam structure. Future research will focus on the use of these ideas for local control of partially assembled structures. In particular, it is natural to simulate large structures by partitioning them into simpler substructures, integrating the substructure dynamics on parallel processors. We plan to develop local controllers along the same lines using substructure-controller optimization and alleviating the deterioration of stability margin due to controller-structure interaction by introducing an appropriate compensator.

First - Order State Estimators

(A, C) observable if and only if $A - KC$ has arbitrary poles

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}) \\ \hat{y} = C\hat{x} \end{cases}$$

Take $\hat{x} = \begin{bmatrix} \hat{q} \\ \hat{\dot{q}} \end{bmatrix}$ & $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$

Then

$$\begin{cases} \dot{\hat{q}} = \hat{\dot{q}} + K_1(y - \hat{y}) \\ \dot{\hat{\dot{q}}} + D_0\hat{\dot{q}} + K_0\hat{q} = B_0u + K_2(y - \hat{y}) \\ \hat{y} = C_0\hat{q} + E_0\hat{\dot{q}} \end{cases}$$

NOTE: If $K_1 = 0$, then $\hat{\dot{q}} = \dot{\hat{q}}$ and

$$\begin{cases} \ddot{\hat{q}} + D_0\dot{\hat{q}} + K_0\hat{q} = B_0u + K_2(y - \hat{y}) \\ \hat{y} = C_0\hat{q} + E_0\dot{\hat{q}} \end{cases}$$

"Natural" Second - Order State Estimators

$$\begin{cases} \ddot{\tilde{q}} + D_0\dot{\tilde{q}} + K_0\tilde{q} = B_0 u + K_3(y - \tilde{y}) \\ \tilde{y} = C_0\tilde{q} + E_0\dot{\tilde{q}} \end{cases}$$

Example:

$$\begin{cases} \ddot{q} + q = u \\ y = q \end{cases} \qquad \text{No velocity measurement}$$

First - Order State Estimator:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = [1 \quad , \quad 0]$$

$$\begin{cases} \dot{\hat{q}} = \hat{\dot{q}} + K_1(y - \hat{y}) \\ \dot{\hat{\dot{q}}} + \hat{q} = u + K_2(y - \hat{y}) \\ \hat{y} = \hat{q} \end{cases}$$

(A, C) observable; so above converges arbitrarily fast by choice of gains K_1 and K_2

Second - Order "Natural" State Estimator:

$$\begin{cases} \ddot{\tilde{q}} + \tilde{q} = u + K_3(y - \tilde{y}) \\ \tilde{y} = \tilde{q} \end{cases}$$

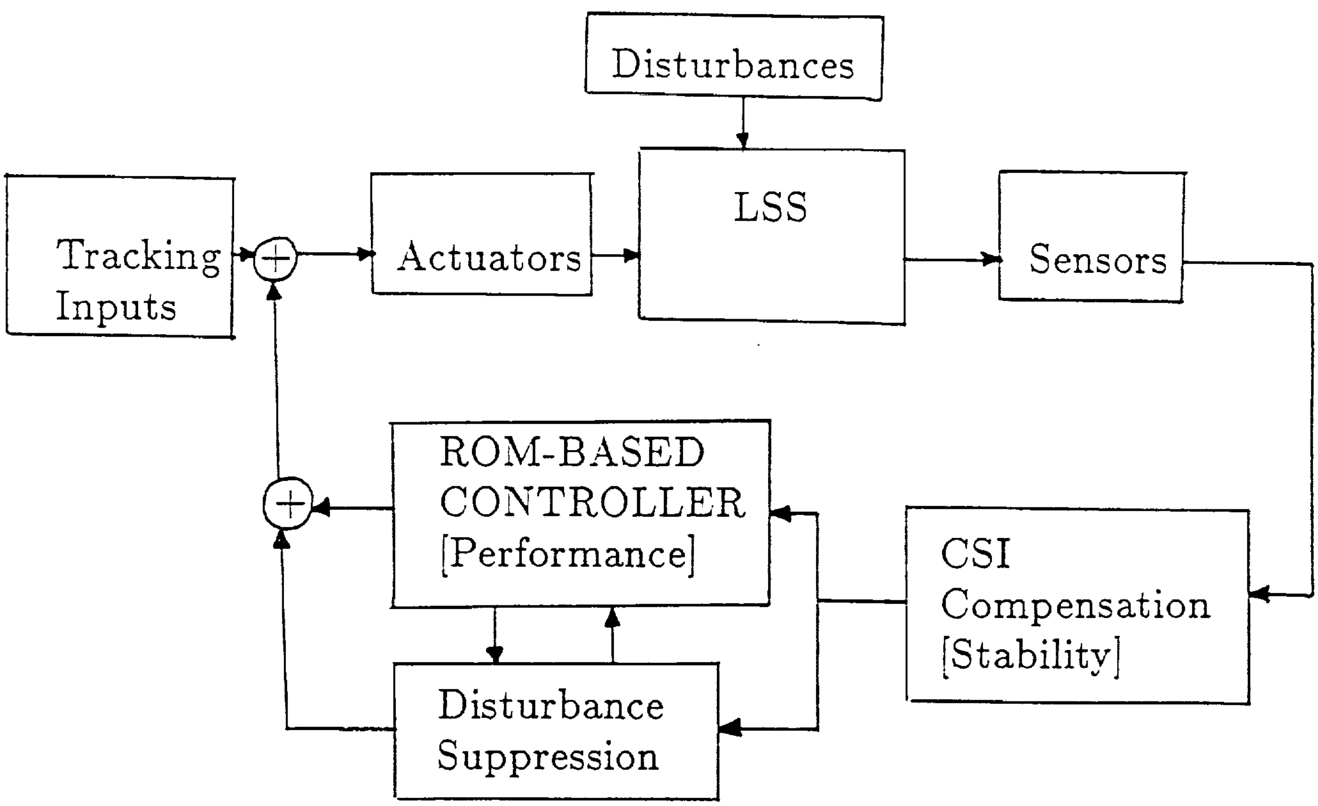
Does not converge for any choice of gain K_3

Current Research

- Distributed Parameter System Theory for Model Reduction and Low-Order Controller Design

- CSI Compensation By Residual Mode Filters

- Numerically Well-Conditioned Methods for Structure/Controller Redesign to Reduce Detrimental CSI



Large Space Structure (Finite Element) Model:

LSS: $M_0\ddot{q} + D_0\dot{q} + K_0 q = B_0 u$

Sensor Outputs: $y = C_0 q + E_0 \dot{q}$

M_0, D_0, K_0 symmetric matrices

(change variables and take $M_0 = I$)

State Space Form:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

$$A = \begin{bmatrix} 0 & I \\ -K_0 & -D_0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ B_0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_0 & E_0 \end{bmatrix}$$

K. Belvin: "If **any** velocity measurement is available anywhere, then a second-order natural state estimator can be built to converge."

Result: (Balas - Quan)

Assume (a) No damping ($D_0 = 0$)

(b) $K_e \equiv K_0 + (C_p^0)^T Q_p C_p^0$ positive definite for some Q_p

(c) (K_e, C_v^0) observable

where $y_p = C_p^0 q$, $y_v = C_v^0 \dot{q}$, and $y = \begin{bmatrix} y_p \\ y_v \end{bmatrix}$.

Then there is **always a convergent** "natural" second-order state estimator:

$$\ddot{\hat{q}} + K_0 \hat{q} = B_0 u + K_p(y_p - \hat{y}_p) + K_v(y_v - \hat{y}_v)$$

$$\hat{y}_p \equiv C_p^0 \hat{q} \quad \text{and} \quad \hat{y}_v \equiv C_v^0 \dot{\hat{q}}$$

where $K_p \equiv (C_p^0)^T Q_p$ and $K_v \equiv (C_v^0)^T Q_v$ with Q_p, Q_v any positive definite matrices.

Example: (Balas - Quan)

$$\begin{cases} \ddot{q}_1 + 2q_1 - q_2 = 0 \\ \ddot{q}_2 - q_1 + q_2 = 0 \\ y_v = \dot{q}_2 \qquad \text{velocity of second mass} \end{cases}$$

NOTE: $K_0 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ and $C_v^0 = [\,0 \quad 1\,]$

First Order Observer:

No damping ($D_0 = 0$)

(K_0, C_v^0) observable if and only if (A, C) observable

therefore, **arbitrarily** fast convergence.

Second - Order "Natural" Observer:

$K_e = K_0$ positive definite and (K_0, C_v^0) observable

$$\begin{cases} \ddot{\hat{q}}_1 + 2\hat{q}_1 - \hat{q}_2 = 0 \\ \ddot{\hat{q}}_2 - \hat{q}_1 + \hat{q}_2 = Q_v(y_v - \dot{\hat{q}}_2)\ ; \quad Q_v \text{positive.} \end{cases}$$

This converges, but the maximum rate is $e^{-0.4t}$

Question: Can every **first-order** state estimator for a LSS be rewritten as a **second-order** state estimator ?

Answer: Yes, but not a "natural" one (Balas - Quan).

Non - Natural Second - Order State Estimators:

$$\begin{cases} \ddot{\varsigma} + D_0\dot{\varsigma} + K_0\varsigma = B_0\tilde{u} + K_3(y - \tilde{y}) + \gamma_c \\ \tilde{y} = C_0\varsigma + E_0\dot{\varsigma} \end{cases}$$

where $\hat{q}$ and $\hat{\dot{q}}$ are **linear** functions of available signals and the corrector term γ_c is also.

Example: $\begin{cases} \ddot{q} + q = u \\ y = q \end{cases}$ No "Natural" Convergent Second - Order State Estimator

Non - Natural Second - Order State Estimator:

$$\begin{cases} \ddot{\varsigma} + \varsigma = \tilde{u} + K_1(y - \tilde{y}) + \gamma_c \\ \tilde{y} = \dot{\varsigma} \end{cases}$$

where $\gamma_c \equiv K_2(v - \varsigma)$; $\dot{v} + \epsilon v = y$; $\dot{\tilde{u}} + \epsilon\tilde{u} = u$; $\hat{q} = \dot{\varsigma}$; $\hat{\dot{q}} = -\varsigma + \tilde{u} + \gamma_c$; $\epsilon > 0$.

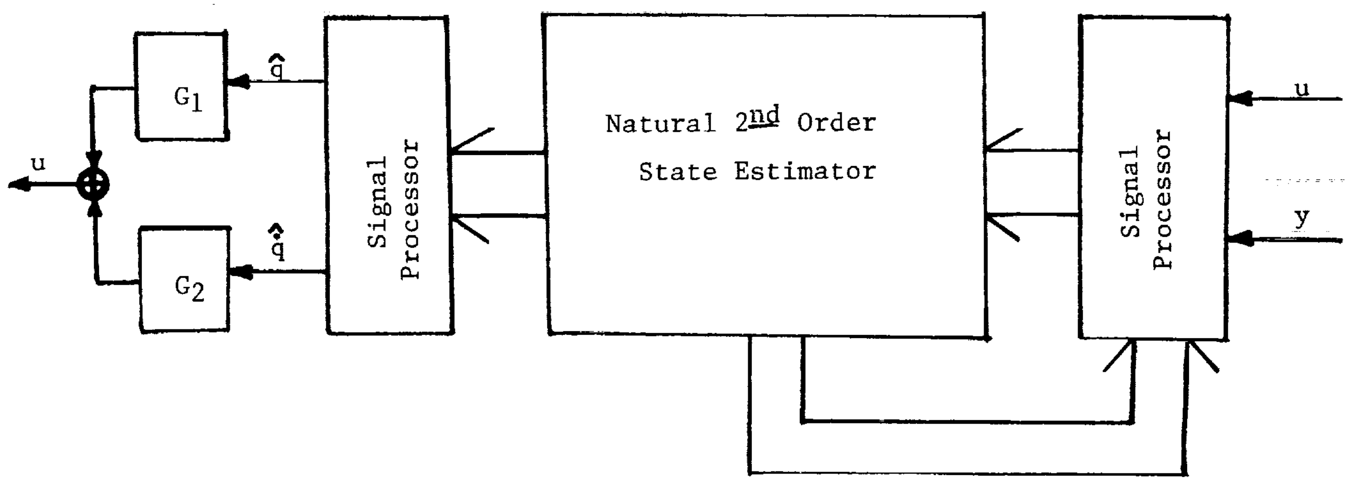
This converges arbitrarily fast.

C-4

Conclusion:

LSS:
$$\begin{cases} \ddot{q} + D_0\dot{q} + K_0 q = B_0 u \\ y = C_0 q + E_0 \dot{q} \end{cases}$$

Controller: $u = G_1\hat{q} + G_2\hat{\dot{q}}$



Advantages:

- ⊙ Controller Based on Second - Order Computer Architecture
- ⊙ Model Reduction Based on Physical Co-ordinate Finite Element Structure Model
- ⊙ Controller Designed for Performance in Physical Co-ordinates; Compensation for CSI induced instabilities added-on, e.g. Residual Mode Filters.